AQA Computer Science A-Level

4.5.2 Number bases

Intermediate Notes

Specification:

3.5.2.1 Number base:

Be familiar with the concept of a number base, in particular:

- decimal (base 10)
- binary (base 2)
- hexadecimal (base 16)

Convert between decimal, binary and hexadecimal number bases.

Be familiar with, and able to use, hexadecimal as a shorthand for binary and to understand why it is used in this way.

Number bases

The same number can be represented in a variety of different ways. Humans use base 10, sometimes called decimal, but we could use binary or hexadecimal.

Decimal (base 10)

Decimal uses the ten digits 0 through to 9 to represent numbers.

Decimal numbers can be denoted with a subscript 10, like so:

$$27_{10}$$

Binary (base 2)

Binary uses only two characters for each digit, either a 1 or a 0.

Binary numbers can be denoted with a subscript 2, like so:

$$10110010_{2}$$

Hexadecimal (base 16)

Hexadecimal uses the digits 0 through to 9 followed by the uppercase characters A to F to represent the decimal numbers 0 to 15.

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Hexadecimal numbers can be denoted with a subscript 16, like so:

$$7E_{16}$$

Conversions

Converting from binary to decimal

You can convert between binary and decimal by using place value headers. Starting with one and doubling the value with each move to the left. For example, the binary number 1010_2 could have place value headers added as follows:

8	4	2	1
1	0	1	0

The binary number could then be converted to decimal by adding together all of the place values with a binary one below them.

$$8 + 2 = 10$$

So the binary number 1010_2 is equivalent to the decimal number 10_{10}.

Converting from decimal to binary

When converting from decimal to binary, you use the same place value headers. Starting from the left hand side, you place a one if the value is less than or equal to your number, and a zero otherwise.

Once you've placed a one, you must subtract the value of that position from your number and continue as before.

Let's say we're converting the number 12 to binary. First, write out your place value headers in powers of two. Keep on going until you've written a value which is larger than your number. For 12, we're going to go up to 16.

16	8	4	2	1

Now, starting from the left, compare the place value to your number. 16 is greater than 12 so we place a 0 under 16.

16	8	4	2	1
0				

Moving to the right, we see that 8 is lower than 12, so we place a 1 under 8.

16	8	4	2	1
0	1			

Because we've placed a one, we have to subtract 8 from 12 to find what's left to be represented. In this case, 12 - 8 = 4.

We move to the right again and find 4, which is exactly what we have remaining to represent, so we place a 1 under 4.

16	8	4	2	1
0	1	1		

Again, because we've placed a one, we have to calculate a new value. 4 - 4 = 0.

Because we've reached 0, all remaining positions can be filled with 0s.

16	8	4	2	1
0	1	1	0	0

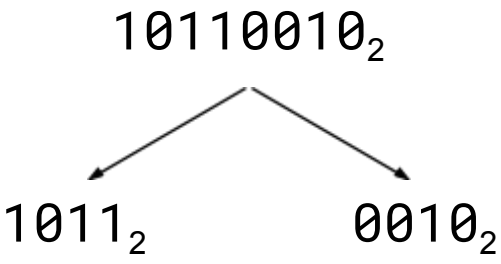
Now that we've placed a 0 or a 1 under each place value, we have our answer. It's acceptable to remove any leading 0s.

$$12_{10} = 1100_2$$

Converting from binary to hexadecimal

In order to convert from binary to hexadecimal, the binary number must first be split into nybbles. A nybble is four binary bits.

For example, the binary number 10110010_2 would be split into two nybbles:



Each binary nybble is then converted to decimal as in the previous example:

8	4	2	1
1	0	1	1

8 + 2 + 1 = 11

8	4	2	1
0	0	1	0

2 = 2

Once each nybble has been converted to decimal, the decimal value can be converted to its hexadecimal equivalent (using the table on page 3) like so:

$$11_{10} = B_{16} \qquad 2_{10} = 2_{16}$$

Finally, the hexadecimal digits are joined together to form a hexadecimal number:

$$10110010_2 = B2_{16}$$

Converting from hexadecimal to decimal (by converting from hexadecimal to binary)

When converting from hexadecimal to decimal, it's usually easiest to go via binary.

First convert your hexadecimal number to binary. Do this by converting each hexadecimal digit to a decimal digit and then to a binary nybble before combining the nybbles to form a single binary number.

$B2_{16}$

Split into hexadecimal digits

B_{16} 2_{16}

Convert hexadecimal to decimal

11_{10} 2_{10}

Convert decimal to binary nybbles

1011_2 0010_2

Combine binary nybbles

10110010_2

Note

When converting from hexadecimal to binary, use this method but stop here.

128	64	32	16	8	4	2	1
1	0	1	1	0	0	1	0

$$128 + 32 + 16 + 2 = 178$$

$$B2_{16} = 178_{10}$$

Converting from decimal to hexadecimal

Converting a number from decimal to hexadecimal is simply the reverse of converting from hexadecimal to decimal.

First convert your decimal number to binary, then split it into decimal nybbles. Each nybble can then be converted to decimal before converting decimal to hexadecimal.

Let's convert 178_{10} to hexadecimal.

First, convert to binary using the method described under "Converting from decimal to binary". If required, add leading 0s to your answer so that you have a whole number of nybbles.

$$10110010_2$$

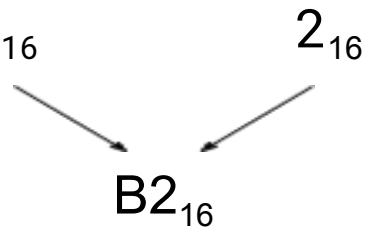
Now split your binary number into nybbles.

$$1011_2 \qquad 0010_2$$

Then convert each nybble to decimal.

$$11_{10} \qquad 2_{10}$$

Lastly, convert decimal to hexadecimal and combine to form a single number.



$$178_{10} = B2_{16}$$